AP CALCULUS SUMMER PACKET Name
DUE THE FIRST DAY OF CLASS.

This material represents an expectation of skills and concepts that are essential in the study of Calculus. If you need a refresher on any of these topics, a google search should lead you to resources. My favorite website is http://www.wolframalpha.com/ which can be used to check your answers.

This assignment must be handwritten on paper. You may not submit electronic versions of this assignment. You are to hand this assignment in on the first day of class. This assignment will serve as the main review for a quiz on this material. The quiz will be administered during the first week of classes. Show your work, answers alone are not sufficient.

Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1, which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \cdot \frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \cdot \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1) $\dfrac{\frac{25}{a}-a}{5+a}$

2) $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3) $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4) $\dfrac{\dfrac{x}{x+1}-\dfrac{1}{x}}{\dfrac{x}{x+1}+\dfrac{1}{x}}$

5) $\dfrac{1-\dfrac{2x}{3x-4}}{x+\dfrac{32}{3x-4}}$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $\left(f\circ g\right)(x) = f(g(x))\ OR\ f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x)=2x^2+1$ *and* $g(x)=x-4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2+1 \\ &= 2(x^2-8x+16)+1 \\ &= 2x^2-16x+32+1 \\ f(g(x)) &= 2x^2-16x+33 \end{aligned}$$

Let $f(x)=2x+1$ *and* $g(x)=2x^2-1$**. Find each.**

6) $f(2) =$ ____________ 7) $g(-3) =$ ______________ 8) $f(a+1) =$ __________

9) $f\left[g(-2)\right] =$ __________ 10) $g\left[f(m+2)\right] =$ ___________ 11) $\dfrac{f(x+h)-f(x)}{h} =$ ______

Let $f(x) = \sin x$ **Find each exactly.**

12) $f\left(\frac{\pi}{2}\right) =$ ___________

13) $f\left(\frac{2\pi}{3}\right) =$ ______________

Let $f(x) = x^2,\ g(x) = 2x + 5,\ \textit{and}\ h(x) = x^2 - 1$. **Find each.**

14) $h\left[f(-2)\right] =$ _______

15) $f\left[g(x-1)\right] =$ _______

16) $g\left[h(x^3)\right] =$ _______

Find $\dfrac{f(x+h) - f(x)}{h}$ **for the given function *f*.**

17) $f(x) = 9x + 3$

18) $f(x) = 5 - 2x$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y = x^2 - 2x - 3$

$\underline{x - \text{int}.\ (\textit{Let}\ y = 0)}$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1\ \textit{or}\ x = 3$

$x - \text{intercepts}\ (-1, 0)\ \textit{and}\ (3, 0)$

$\underline{y - \text{int}.\ (\textit{Let}\ x = 0)}$

$y = 0^2 - 2(0) - 3$

$y = -3$

$y - \text{intercept}\ (0, -3)$

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Find the x and y intercepts for each.

19) $y = 2x - 5$

20) $y = x^2 + x - 2$

21) $y = x\sqrt{16 - x^2}$

22) $y^2 = x^3 - 4x$

Use substitution or elimination method to solve the system of equations.
Example:

First equation should be $x^2+y^2-16x+39=0$

$$x^2 + y - 16x + 39 = 0$$
$$x^2 - y^2 - 9 = 0$$

Elimination Method

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \text{ and } x = 5$

Plug $x=3$ *and* $x = 5$ into one original

$3^2 - y^2 - 9 = 0$ $\quad$ $5^2 - y^2 - 9 = 0$

$-y^2 = 0$ $\quad$ $16 = y^2$

$y = 0$ $\quad$ $y = \pm 4$

Points of Intersection $(5,4)$, $(5,-4)$ *and* $(3,0)$

Substitution Method

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ (*The rest is the same as previous example*)

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \text{ or } x - 5$

Find the point(s) of intersection of the graphs for the given equations.

23) $x + y = 8$
$4x - y = 7$

24) $x^2 + y = 6$
$x + y = 4$

<u>Interval Notation</u>

25) Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1, 7)$	
		 **8**

Solve each equation. State your answer in BOTH interval notation and graphically.

25) $2x - 1 \geq 0$

26) $-4 \leq 2x - 3 < 4$

27) $\frac{x}{2} - \frac{x}{3} > 5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

28) $f(x) = x^2 - 5$

29) $f(x) = -\sqrt{x+3}$

30) $f(x) = 3\sin x$

31) $f(x) = \dfrac{2}{x-1}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x) = \sqrt[3]{x+1}$	Rewrite f(x) as y
$y = \sqrt[3]{x+1}$	Switch x and y
$x = \sqrt[3]{y+1}$	Solve for your new y
$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3 = y+1$	Simplify
$y = x^3 - 1$	Solve for y
$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation

Find the inverse for each function.

32) $f(x) = 2x + 1$

33) $f(x) = \dfrac{x^2}{3}$

<u>Equation of a line</u>

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: y = c (slope is 0)

34) Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

35) Determine the equation of a line passing through the point (5, -3) with an undefined slope.

36) Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

37) Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

38) Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

39) Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

40) Find the equation of a line passing through the points (-3, 6) and (1, 2).

41) Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Angles in Standard Position

42) Sketch the angle in standard position.

a. $\frac{11\pi}{6}$ b. 230° c. $-\frac{5\pi}{3}$ d. 1.8 radians

Reference Triangles in a Unit circle

43) Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

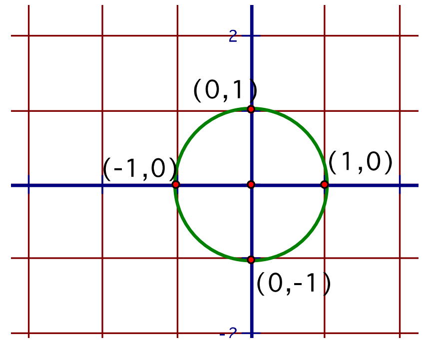
a. $\frac{2}{3}\pi$ b. 225°

c. $-\frac{\pi}{4}$ d. 30°

Unit Circle

You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

Example: $\sin 90^\circ = 1$ $\cos\frac{\pi}{2} = 0$



44) $a.)\ \sin 180^\circ$ $b.)\ \cos 270^\circ$ $c.)\ \sin(-90^\circ)$ $d.)\ \sin\pi$ $e.)\ \cos 360^\circ$ $f.)\ \cos(-\pi)$

Trigonometric Equations:

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

45) $\sin x = -\frac{1}{2}$

46) $2\cos x = \sqrt{3}$

47) $\cos 2x = \frac{1}{\sqrt{2}}$

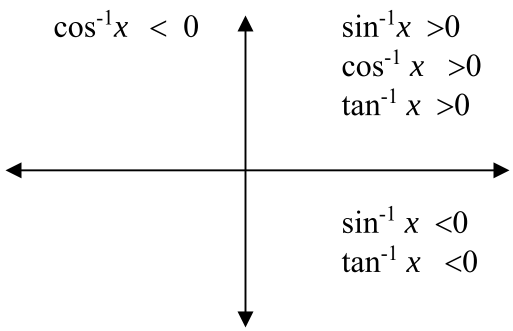
48) $\sin^2 x = \frac{1}{2}$

Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of ways:

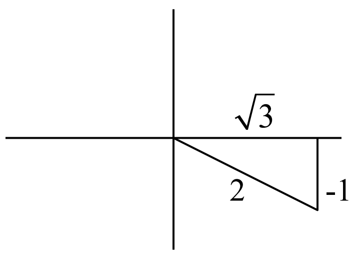
$$\arcsin(x) \qquad \sin^{-1}(x)$$

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:
Express the value of "y" in radians.

$y = \arctan\frac{-1}{\sqrt{3}}$ Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, y = $-\frac{\pi}{6}$ so that it falls in the interval from

$$\frac{-\pi}{2} < y < \frac{\pi}{2} \qquad \text{Answer: } y = -\frac{\pi}{6}$$

For each of the following, express the value for "y" in radians.

49) $y = \arcsin\frac{-\sqrt{3}}{2}$

50) $y = \arccos(-1)$

51) $y = \arctan(-1)$

Vertical Asymptotes (No Holes)

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

52) $f(x)=\dfrac{1}{x^2}$ 53) $f(x)=\dfrac{x^2}{x^2-4}$ 54) $f(x)=\dfrac{2+x}{x^2(1-x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

55) $f(x)=\dfrac{x^2-2x+1}{x^3+x-7}$ 56) $f(x)=\dfrac{5x^3-2x^2+8}{4x-3x^3+5}$ 57) $f(x)=\dfrac{4x^5}{x^2-7}$

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Mixed Practice

Factor completely.

58. x^5+11x^3-80x

59. $(x-3)^2(2x+1)^3+(x-3)^3(2x+1)^2$

60. $2x^2+50y^2-20xy$

Solve the following inequalities by factoring and making sign charts.

61. $x^2-16>0$

62. $x^2+6x-16>0$

63. Rewrite $\frac{1}{2}\ln(x-3)+\ln(x+2)-6\ln x$ as a single logarithmic expression.

64. Use the table to calculate the average rate of change from $t=1$ to $t=4$.

t	0	1	2	3	4
$x(t)$	8	7	5	1	2

65. Sketch a graph of the piecewise function $f(x)=\begin{cases}-x^2, & -2\le x<1\\ -2, & x=1\\ 3x+5, & 1<x\le 3\end{cases}$.